Code: CT22 **Subject: DISCRETE MATHEMATICAL STRUCTURES**
Time: 3 Hours **Max. Marks: 100**

MARCH 2011

NOTE:
- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**

Q.1 a. Prove that $p \rightarrow (q \rightarrow r)$ and $(p \wedge \neg r) \rightarrow \neg q$ are logically equivalent.

b. In a survey, 2000 people were asked whether they read 'India Today' or 'Business Times'. It was found that 1200 read 'India Today', 900 read 'Business Times' and 400 read both. Find how many read at least one magazine and how many read neither.

c. What is the minimum number of students required in a class to be sure that at least 6 will receive the same grade if there are five possible grades A, B, C, D and F?

d. Determine whether the following is a valid argument:
I am happy if my program runs. A necessary condition for the program to run is it should be error free. I am not happy. Therefore the program is not error free.

e. Let m be a positive integer with m>1. Determine whether or not the following relation is an equivalent relation: $R = \{(a,b) | a \equiv b \pmod{m}\}$

f. Define Euler path, Euler circuit, Hamiltonian path and Hamiltonian circuit.

g. Design a DFA for recognizing identifiers, unsigned integers and unsigned real numbers with fractions **(7 × 4)**

Q.2 a. Consider the set A = {2, 7, 14, 28, 56, 84} and the relation $a \leq b$ if and only if a divides b. Give the Hasse diagram for the poset $(A, \leq)$.

b. Let L be a distributive lattice. Show that
if $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$, then $x = y$ for same a. **(9+9)**

Q.3 a. Translate the following into logical notation. Let the universe of discourse be the real numbers.

(i) For any value of x, x^2 is non negative.
(ii) For every value of x, there is some value of y such that x.y=1.
(iii) There are positive values of x and y such that x.y>0
(iv) There is a value of x such that if y is positive, then x+y is negative.

b. Minimize the following Boolean expression and give its DNF.

$$F(A, B, C, D) = \pi(0, 1, 4, 5, 8, 12, 13, 14, 15)$$

c. Determine the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. **(8+7+3)**

Q.4 a. Convert the following proposition into polish prefix notation.
$(p \rightarrow (q \rightarrow r)) \equiv (\neg (p \vee r) \wedge \neg q)$

b. If $A \subseteq C$ and $B \subseteq D$, prove that $A \times B \subseteq C \times D$ **(9+9)**

Q.5 a. If G(V,E) is a simple connected planar graph, then $3v-6 \geq e$; where e is the total number of edges and v is the total number of vertices in the graph G. Use this to prove that complete graph K_5 is non planar.

b. Let A = {1, 2, 3, 4} and, R = {(1, 1), (1, 4), (2, 1), (2, 2), (3, 3), (4, 4)}. Use Warshall's algorithm to find the transitive closure of R. **(8+10)**

Q.6 a. What is minimum spanning tree of a graph? Execute Prim's algorithm to find minimum spanning of the following graph (Fig. 1).

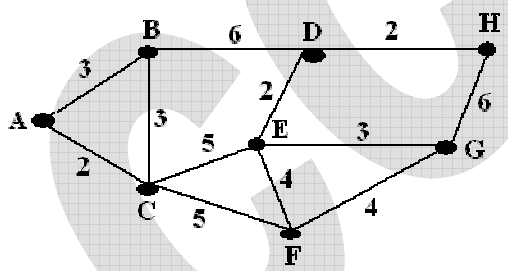

Fig.1

b. Let A is a set of all positive real numbers and B is a set of all real numbers. Let f be a function $f:A \rightarrow B$ defined as $f(x)=\log_e x$. Show that f is one to one and onto function. **(12+6)**

Q.7 a. Let L be a language over {0, 1} such that each string starts with a 0 and ends with a minimum of two subsequent 1's. Construct,

(i) the regular expression to specify L.
(ii) a finite state automata M, such that M (L) = L.
(iii) a regular grammar G, such that G(L) = L.

b. Consider the following productions:

$$S \rightarrow aB \mid bA$$

$$A \rightarrow aS \mid bAA \mid a$$

$$B \rightarrow bS \mid aBB \mid b$$

Then, for the string a a a b b a b b b a, find the
(i) the leftmost derivation.
(ii) the rightmost derivation. **(9+9)**